Good States, Bad States:
What Do Options Tell Us About Schizophrenic Behavior of Mr. Market and What Can We Do About It?

This Version: November 25, 2020

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Abstract

Option prices theoretically encapsulate participants' expectations about good state (bullish) and bad state (bearish) market outcomes. By using a mixture of distributions and reasonable assumptions, the authors extract time series of expected returns, volatilities and mixture probabilities of these outcomes surrounding the current US elections. The bimodality of asset return distributions suggests important modifications for asset allocation and risk management.

Key Takeaways

- Rather than being unimodal, asset price distributions reflect a mixture of good and bad states. The probability of being in these states changes with time and during important events.
- During elections, the bimodality and the spread of the mixture distributions increases as a function of increased uncertainty.
- Since a mixture of distributions can exhibit fatter tails and negative skew, the mixture approach requires asset allocation adjustments and hedging to manage risks.

Investing in financial markets requires participants to become comfortable with uncertainty. Especially during important events and episodes, such as elections, key data releases, or catastrophic occurrences, the perceived uncertainty reflected in market prices tends to rise. Given the impact of low probability, high severity events on portfolios, it is important for investors to understand the likelihood and probability of potential outcomes and position portfolios accordingly. Even though in the long run, markets reflect fundamentals, and certainly the stock market reflects long-term growth of the economy, in the short run, e.g. over a few days, and even over a month, Mr. Market can change his mind rapidly and without warning. As Warren Buffett remarked in his 1987 letter to shareholders, quoting his mentor Ben Graham:

Even though the business that the two of you own may have economic characteristics that are stable, Mr. Market's quotations will be anything but. For, sad to say, the poor fellow has incurable emotional problems. At times he feels euphoric and can see only the favorable factors affecting the business. When in that mood, he names a very high buy-sell price because he fears that you will snap up his interest and rob him of imminent gains. At other times he is depressed and can see nothing but trouble ahead for both the business and the world. On these occasions, he will name a very low price, since he is terrified that you will unload your interest on him.

Options markets in principle encapsulate the views of all market participants, aggregating in prices the mixture of both pessimists and optimists. Additionally, options markets not only allow investors to extract information about the uncertainty prevalent in the markets, but they also allow investors to use this information to buy or sell the options themselves. This feedback mechanism can thus potentially influence further estimation of the uncertainty. Once the options markets themselves reach a locally stable point, it is possible to quantify the uncertainty in the form of implied probability distributions, and investors can overlay their own probabilistic forecasts on top of the market pricing to take advantage of perceived mispricing by market participants in the aggregate.

While the standard approach follows extracting a uni-modal implied distribution from option prices, our approach differs by imposing a very simple bimodal structure on the markets, and thus extracting the market's probability of being in a "good" state or in a "bad" state from locally stable options prices before and after important events. In the good state the markets are posited to have positive expected returns, and in the bad state the markets have a negative expected return. The probability of being in the states is the quantification of the market's perception of how likely we are to end up in one of these states.

We hasten to add that none of the techniques in this paper are new or original. However, we think that the simple yet revealing approach we utilize in the context of recent market events is of critical importance for investor asset allocation decisions. Indeed, many techniques have been developed (see e.g. Breeden and Litzenberger [1978]) to extract the risk-neutral probability distribution of outcomes from option prices. In this paper, we go one step further: we take option prices over various recent events and fit them to a bimodal distribution formed as a mixture of two lognormal distributions. This formulation requires the estimation of five parameters corresponding to the mean and standard deviation of the two lognormal distributions and the mixture probability. With this simple yet parsimonious formulation, we can ask three main questions:

1. What do the component distributions tell us about the expectations of risk and return in “good" (bull market), and “bad" (bear market) states? How do these expectations evolve with time as a function of major market events?
2. Does the probability of being in bull or bear states change significantly over time, especially during market crises and other significant market events?
3. Does the knowledge gained about the mixtures forming the actual implied distribution change optimal asset allocation and risk management techniques?

Our main findings regarding the first question are that the shape of the implied return distributions change significantly during normal and stressed market situations, and in particular the uncertainty in both distributions changes significantly during market stresses. Regarding the second question, we find that the probability of being in bull states and bear states during the period that our options data covers is relatively stable, which was a somewhat surprising result given the general belief both in the practitioner and behavioral finance literature that variation of subjective probabilities play the dominant role in shaping perceptions of risk and return. Our answer to the third question is that armed with the changing shape and mixture of the bull and bear states, investors can dynamically improve their risk management posture.

Our purpose is to use this simple framework to explain the risk perception of market participants corresponding to two very distinct states. As discussed elsewhere, the possibility of bimodality in asset returns can significantly impact both the optimal asset allocation of investors, and the need for hedging (Bhansali [2013]). We have also found that when the probability of one type of event (good or bad) starts to approach unity as implied by the options markets, it can imply overconfidence in aggregate, and the market as a whole is unprepared for a reversal in the consensus.

Extracting Bull and Bear Market Distributions

In this section, we describe our simple but parsimonious model. From there, we present the main results for the S&P500 in special periods of interest, such as the financial crisis and the recent U.S. Presidential elections.

In our model, the expected value of an asset, S, is the probability weighted average of its future payoffs at T discounted back to the present time

$$S = e^{-rT} \int f(S_T) S_T dS_T$$

The probability density function f is estimated from option prices by taking the second derivative of call option prices with respect to strike (Breeden and Litzenberger [1978]):

$$f(K) = e^{rT}\frac{d^2C(K)}{dK^2}$$

To capture potential bimodality of the distribution, a mixture of lognormals can be used to model f (Melick and Thomas [1997]). In order to split the probability distribution into two distinct states, i.e. "good" and "bad", or "bull" and "bear", we force f to be a mixture of two lognormals with mixing probability p. We take $f_B(K,p)$ to take the form:

$$f_B(K,p) = p\,\frac{1}{K\sigma_A\sqrt{2\pi}}\exp\left(\frac{-\left(\ln\left(\frac{K}{S}\right)-\mu_A\right)^2}{2\sigma_A^2}\right) + (1-p)\frac{1}{K\sigma_B\sqrt{2\pi}}exp\left(\frac{-\left(\ln\left(\frac{K}{S}\right)-\mu_B\right)^2}{2\sigma_B^2}\right)$$

$\boldsymbol{\theta} = (p, \mu_A, \sigma_A, \mu_B, \sigma_B)'$ are the parameters of the distribution to be estimated. We refer to μ_A as good or bull state expectations, σ_A as bull state volatility and correspondingly μ_B, σ_B as the bad or bear state parameters. The probability of being in the bull state is denoted by p.

The fitting process proceeds as follows. Similar to Shimko [1993], we first use traded options prices to derive Black-Scholes implied volatilities and fit them to a quartic polynomial. This continuous function of the implied volatility curve is next used in the Black-Scholes formula to generate a continuous curve of option call prices. Finally, finite differencing is applied twice to the call prices to locally approximate the second derivative around the strikes of the original traded options. To fit the parameters of the mixture, we use standard nonlinear least squares.

Quantifying Market Bimodality

The daily set of European option contracts on the S&P500 index (SPX) are collected from OptionMetrics. The full data period for this paper starts in 01/01/1996 and ends in 10/31/2020, and covers a couple of bull markets, the financial crisis and recovery, the taper tantrum, as well as the recent COVID-19 crisis. From the list of options contracts, only out of the money options are considered since they are most frequently traded. All contracts with zero bids are also removed.

In this section, we estimate the time varying parameters $\widehat{\boldsymbol{\theta}}_{\boldsymbol{t}} = (\hat{p}, \hat{\mu}_A, \hat{\sigma}_A, \hat{\mu}_B, \hat{\sigma}_B)_t{}'$ on a monthly basis using implied volatilities one month out. The subscripts A refer to good states and the subscript B refers to bad states.

Exhibits 1-3 plot the parameters over time. We observe that bear market volatility perception tends to be higher than bull market volatility, i.e. $\sigma_A < \sigma_B$, which is clearly related to a high put skew on the S&P500 index, i.e. the market rationally expects volatility to be higher in bad states.

In Exhibit 1 we display the time series of the bull state probability over the next month. We find that this probability is on average around 63% (see summary statistics in Exhibit 4 below), with an interquartile range of between 62% and 64%. The low of the probability of being in the bull state is around 46%, and the high is around 73% over the full historical sample. Surprisingly, while the low in the bull probability corresponded with the "Great Financial Crisis of 2008-2010 (GFC), the probability remained quite stable in the current "Global Virus Crisis" of spring 2020 (GVC).

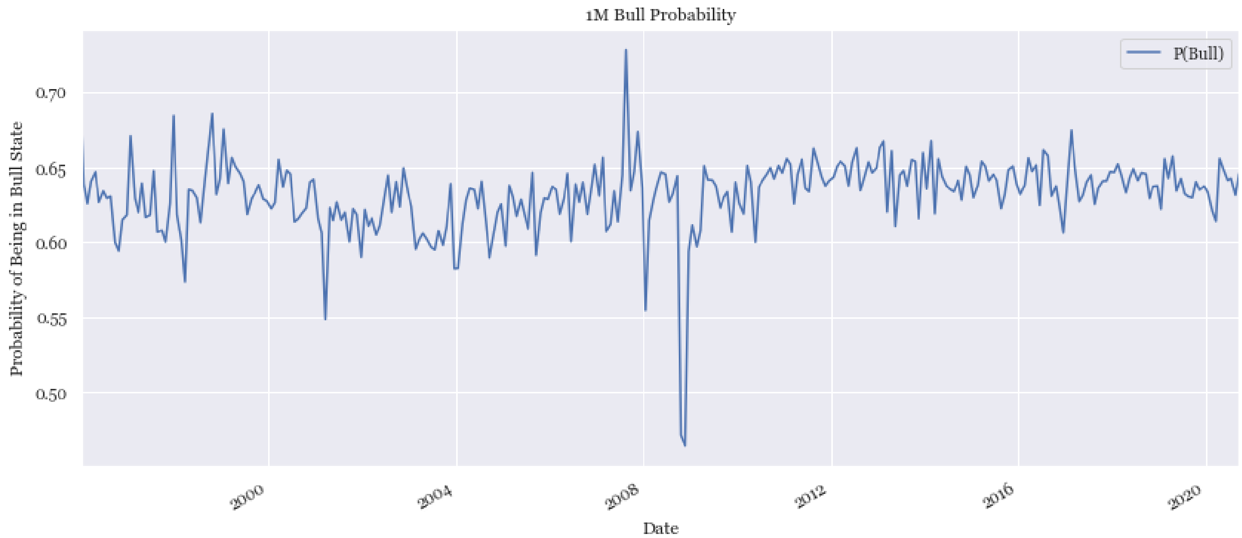

Exhibit 1: Time Varying Bull State Probability
Source: LongTail Alpha, Bloomberg, OptionMetrics

In Exhibit 2, the effect of the bimodal structure of the model becomes apparent. In both the episodes of the global financial crisis and the recent global virus crisis, the expected bull state returns spiked up, whereas the expected bear state returns went significantly more negative. This illustrates that conditional on one of these states occurring, the market perceives larger than average absolute returns. This is a sign that both left and right tails become more likely during these crises. Surrounding these extraordinary market events, the market's overall expectations of such fat tails realizing are usually quite good. For instance, even though during both the GFC and GVC it was hard to ascertain the ultimate timing and unfolding of the chain of events, nonetheless, the fact that a deep selloff was followed by a substantial rally is some validation that the market's estimation of conditional expected returns was quite good.

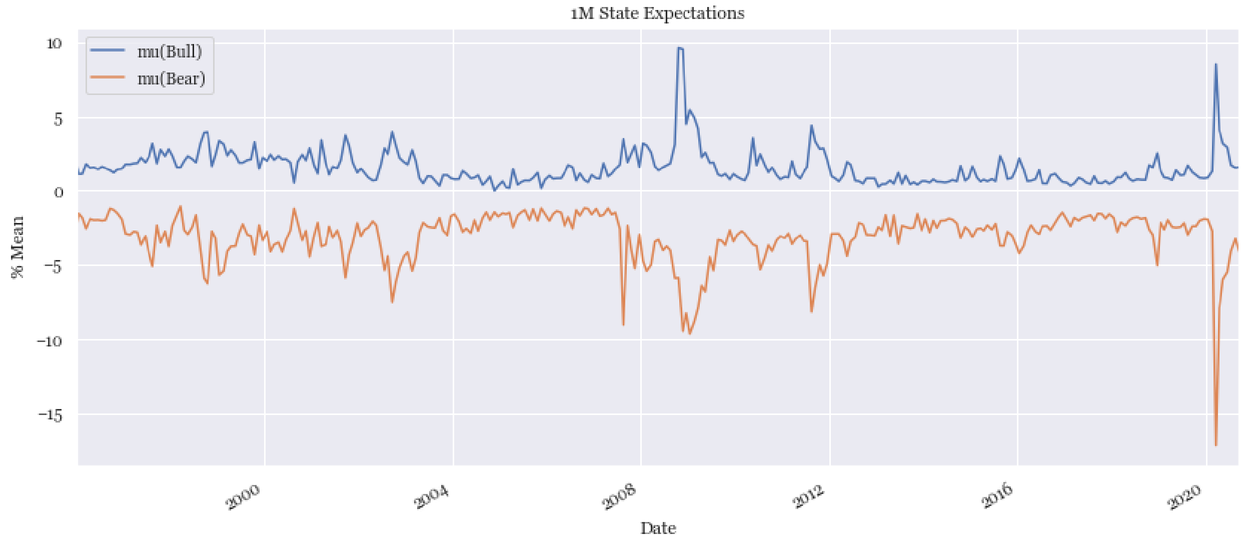

Exhibit 2: Time Varying State Return Expectations
Source: LongTail Alpha, Bloomberg, OptionMetrics

In Exhibit 3, we display the volatility of the bad state distribution and the volatility of the good state distribution over history, along with the VIX index (normalized to monthly horizon to match the estimated volatilities). A couple of observations stand out. First, the increase in volatility is tracked by the VIX quite well. However, the VIX spikes are predominantly capturing the spike up in the bear state volatility. In other words, the equity index options market, at one month horizon, is predominantly capturing downside crash risk. Thus, selling the VIX (via futures), has been roughly equivalent to selling downside crash risk. This is consistent with the idea that the volatility risk premium in the VIX is predominantly a crash risk premium. Any VIX contingent risk management strategy then is largely a downside risk management strategy if historical data is reliable.

Interestingly, the mixture parameter p is less variable over time and stays roughly 63% over the entire period, indicating a relatively constant higher probability of being in a bull market. This was somewhat surprising to us, since much recent work in behavioral finance suggests that systematic mis-estimation of tail probabilities could be behind the option skew. It is possible that our model in this paper is too simplistic, or it is possible that over this period the market consistently believed that eventually all selloffs are temporary and will turn into rallies. The sample set has coincided with large central bank stimulus targeting asset prices, and recent action by central banks to boost liquidity and thus equity markets proposes a mechanism by which this belief could have been reaffirmed.

We conclude from this simple analysis that for the equity markets, the divergence between bull and bear states is primarily quantified in terms of the consequences, i.e. the expected returns and volatilities of the states, rather than the probabilities of the states.

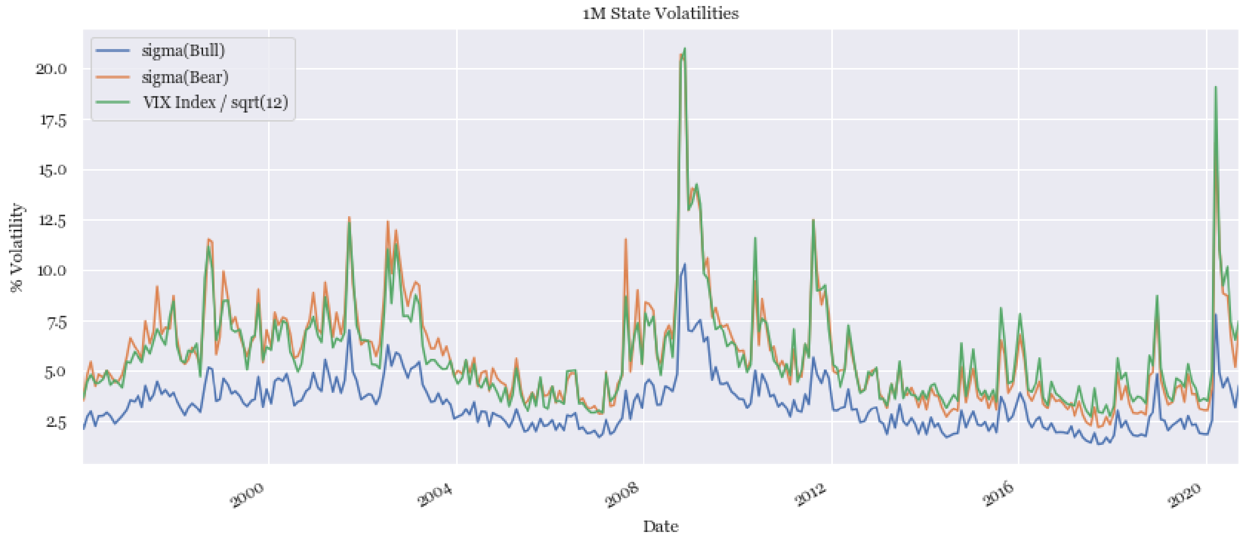

Exhibit 3: Time Varying State Volatilities
Source: LongTail Alpha, Bloomberg, OptionMetrics

Exhibit 4 summarizes the statistics for the five parameters over the long-term history. We have split the exhibit into two parts. The upper part shows the statistics for all periods combined, whereas the lower part shows the statistics only for the periods where the VIX was over 25, which we call "crisis" periods. The crisis period observations corresponded to roughly one-fifth, or twenty percent, of the total observations. The main observation from this table is that even though the mean probability of switching between states does not change very much, the conditional expected returns and volatilities in the crisis period roughly double in absolute value when compared to the periods that include all the data.

	P(Bull)	mu(Bull)	sigma(Bull)	mu(Bear)	sigma(Bear)
count	298	298	298	298	298
mean	0.63	1.56	3.31	-3.10	5.82
std	0.02	1.22	1.31	1.73	2.69
min	0.46	-0.01	1.35	-17.18	2.18
25%	0.62	0.77	2.37	-3.63	3.83
50%	0.63	1.21	3.10	-2.69	5.20
75%	0.65	1.94	3.96	-2.00	6.90
max	0.73	9.62	10.27	-1.05	20.68

(a) Entire sample

	P(Bull)	mu(Bull)	sigma(Bull)	mu(Bear)	sigma(Bear)
count	61	61	61	61	61
mean	0.63	3.17	5.16	-5.52	9.75
std	0.04	1.64	1.33	2.23	2.88
min	0.46	1.09	3.57	-17.18	6.50
25%	0.62	2.26	4.33	-5.98	7.96
50%	0.64	2.87	4.81	-5.04	8.95
75%	0.64	3.42	5.43	-4.13	10.57
max	0.73	9.62	10.27	-2.74	20.68

(b) Crisis periods: VIX Index > 25

Exhibit 4: Parameter Summary Statistics For Entire Sample Since 2000-2020
Source: LongTail Alpha, Bloomberg, OptionMetrics

To summarize all these results, Exhibit 5 displays the probability densities of the parameters fit over two periods: the entire sample and during crisis periods when the VIX index is greater than 25. As discussed, during the crisis periods, bull state and bear state returns are higher and lower respectively than they are over the whole sample, i.e. the disagreement between mean returns in these states increases. Additionally, the standard deviation of the returns increases, i.e. investors become more uncertain about the levels of these returns.

Interestingly, over the twenty yearlong dataset, we find that whereas the probability distribution of the switching parameter (between good states and bad states) fattens somewhat during crises, its mean remains largely unchanged. Thus, despite increased uncertainty on which state the future will bring, the odds are that we end up in the bull state. This has consequences for portfolio construction and risk management. Looking at the second panel of Exhibit 5, this analysis suggests that periods of heightened uncertainty are periods where excess liquidity can be deployed for higher ex-ante returns. In other words, crisis creates opportunities for higher ex-ante returns as long as one does not fall into the bad state of low or negative returns permanently.

One way to achieve this asymmetric outcome is to insure against crises using tail hedging or other risk mitigation strategies, so that the hedges allow one to allocate liquidity during crises. The last panel of Exhibit 3 also suggests that while implied bear state volatilities are generally higher than bull state volatilities, the uncertainty in bear state volatility distributions is also higher, i.e. the probability distribution of bear state volatility is fatter than the bull state distribution. Further, the bear state volatility distribution, especially during crises, has a very fat right skew, which points to heightened perceived tail risk in crises events. This observation has important consequences for the construction of risk premium portfolios. To the extent that risk premium portfolios are exposed to shocks to volatility, they need to be able to not only withstand, but actually to increase their exposure to risk premium harvesting strategies during crises. As the events of 2020 illustrated, however, many risk premium harvesting strategies were actually forced to de-risk during the sharp market selloff of early 2020, and in the process missed the opportunity to benefit from the increased risk premium realized in the aftermath of the selloff. To us, it seems that one practical way to avoid such an outcome in the future is to approach the risk premium portfolio construction problem as one would do in the insurance market, i.e. for every insurance policy sold, also consider purchasing some re-insurance to cut off fat tail risk when premiums are low and the markets are in a good state. This would prevent forced liquidation, and further create even better opportunities in periods when there is a crisis and the volatility risk premium is much higher than normal.

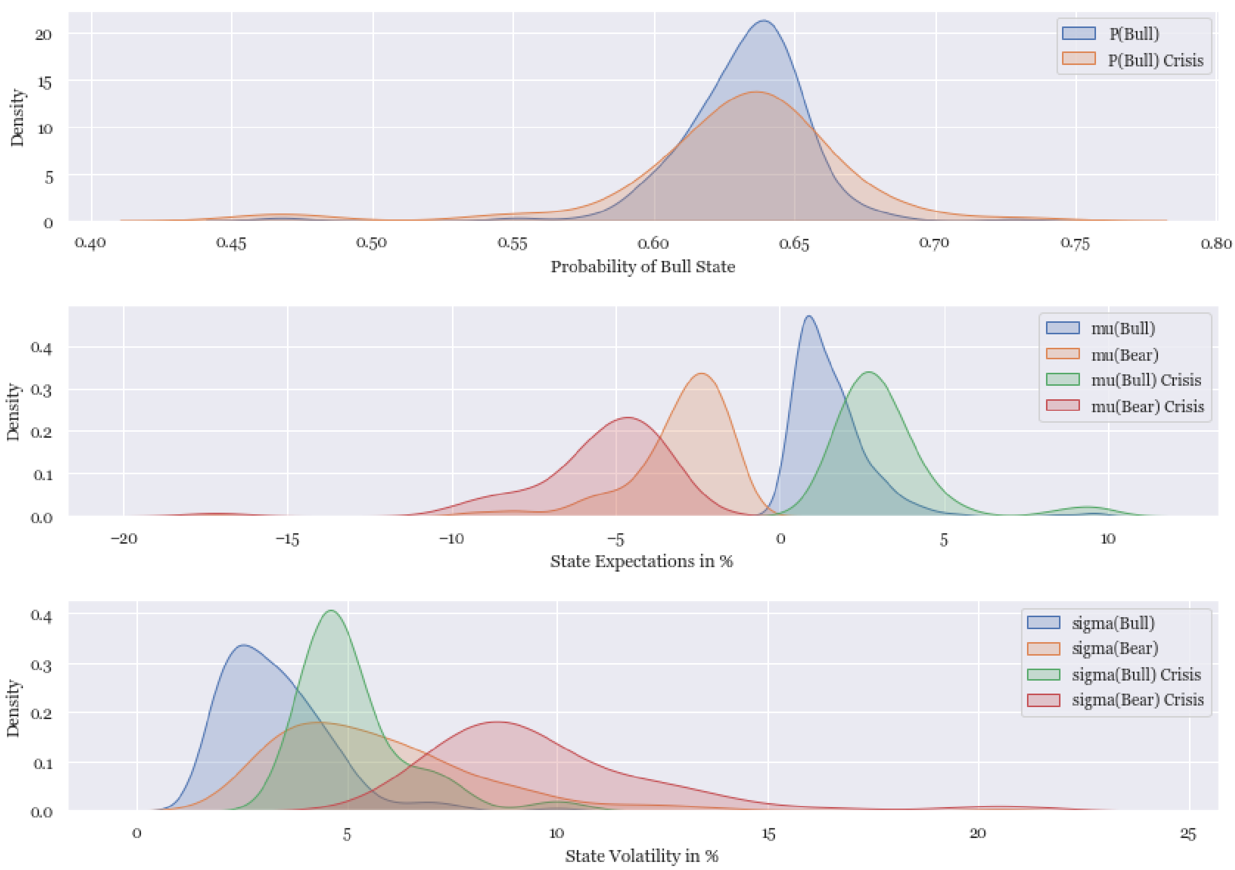

Exhibit 5: Parameter Densities Over Entire Sample and Crisis Periods From 2000 to 2020

Source: LongTail Alpha, Bloomberg, OptionMetrics

Historical Analysis of Mixing From The 2016 U.S. Election

The 2016 Presidential election was an event for the record books. Prior to the election, there was general consensus that Hillary Clinton would win the election. The possibility of a Trump win coming into Election Day was low, and conditional on a Trump win, the market was expected to fall sharply. As the results started to come in on election night showing a possible Trump win, the S&P index futures markets fell to their maximum downside limit. However, as the full impact of market friendly tax proposals under a Trump administration became known, the market rallied sharply, and continued an almost 60% ascent over the next few years.

Following the methodology described earlier, the binary distribution is fit on dates pre and post the 2016 U.S. election. The dates chosen were 11/4/2016 and 11/11/2016, while the actual election took place on 11/8/2016. Exhibit 6 below summarizes the fitted parameters of the binary mixture distributions using implied volatilities for one month options.

	P(Bull)	mu(Bull)	sigma(Bull)	mu(Bear)	sigma(Bear)
Date					
11/4/2016	0.59	2.34	2.87	-2.83	5.35
11/11/2016	0.63	1.05	2.00	-1.94	3.86

Exhibit 6: 1M Parameters Pre and Post 2016 Presidential Election
Source: LongTail Alpha, Bloomberg, OptionMetrics

The parameters fit the narrative for this event very intuitively. Before the U.S. election date, investors naturally were worried about possible outcomes and as such perceived risks were generally higher, being reflected in much higher volatilities over the next month for both bull and bear states (see exhibit 6). On the other hand, they were also confident that candidate Clinton was going to win the election, which at the time was perceived as the outcome more favorable to the markets. As such, forward looking returns were bullish. The night of the election, there were huge swings in markets as nervous investors lost their risk appetite as returns showed the possibility of a Trump win. A week after the election, the markets reflected optimism in the pro-growth policies of a Trump administration and a Republican Congress. This reduced the bimodality as both bullish and bearish sentiment retreated to normal levels. Exhibit 7 illustrates the fitted mixture densities pre and post-election. The vertical lines represent the location of the spot price relative to the strikes pre and post-election.

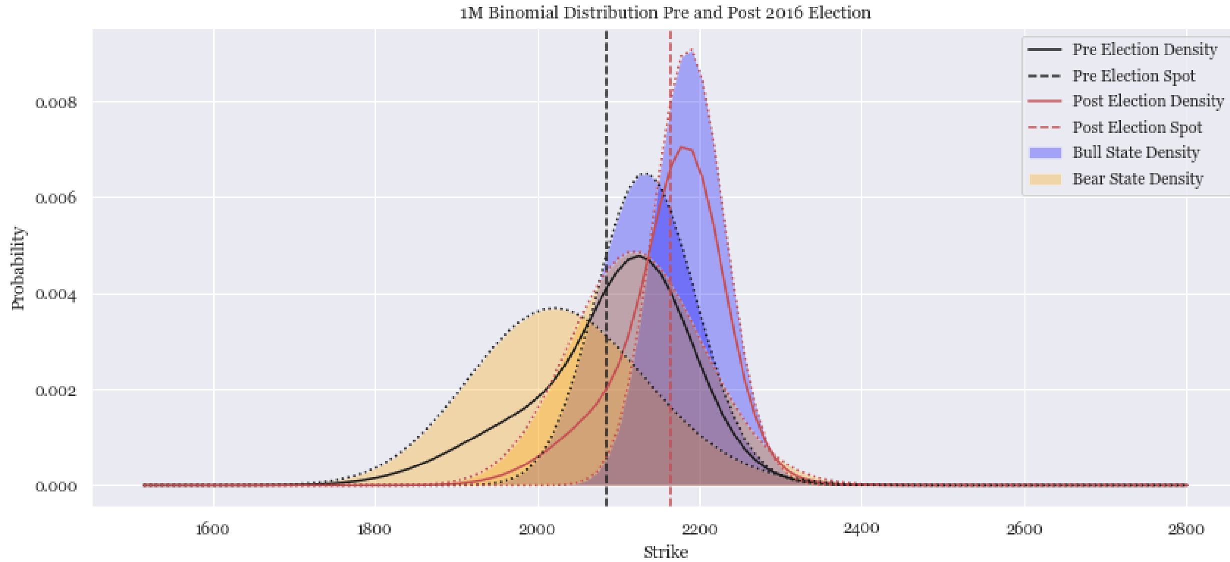

Exhibit 7: 1M Densities Pre and Post 2016 Presidential Election
Source: LongTail Alpha, Bloomberg, OptionMetrics

Real Time Mixing: The 2020 U.S. Election

At the time of this writing, Presidential elections in the U.S. for the 2020 were just concluded, but the results were not officially accepted by the incumbent, Donald Trump. The general perception in the markets is that the Democratic candidate, Joe Biden, is less business and market friendly. Muddying the analysis is the fact that as of this writing (mid November 2020), Congressional elections are also somewhat undetermined. The general consensus in the markets is that if both the House and the Senate turn Democrat, the Trump tax friendly initiatives will be swiftly overturned, and this would be negative for the companies who have benefited from lower taxes. On the other hand, a split Congress would make any tax changes much harder to implement, so "status quo", i.e. not large changes, would be more market friendly.

The parameter estimates are displayed in Exhibit 8. As we can see the probability of being in the bull and bear states has not changed. This could be partly due to the status quo described above, or could be due to the fact that the stock market is increasingly reliant on the aid from the Federal Reserve. Looking at the volatility estimates, we see that both bull and bear state volatilities have collapsed in the aftermath of the election, but the bear state volatility has seen a larger percentage decline than the bull state volatility. The conditional negative expected bear state return has also fallen by over half, accompanied by a fall in the bear state uncertainty. This suggests to us that the passing of the election has taken out the election event risk, with the markets left generally comfortable that the stimulus action from the Fed, and potential future fiscal action is on balance good for the stock market.

	P(Bull)	mu(Bull)	sigma(Bull)	mu(Bear)	sigma(Bear)
Date					
10/30/2020	0.64	3.82	4.98	-7.88	11.21
11/11/2020	0.64	1.73	3.66	-3.76	6.02

Exhibit 8: 1M Parameters Pre and Post 2020 Presidential Election
Source: LongTail Alpha, Bloomberg, OptionMetrics

Finally, as displayed in exhibit 9, we can see that both state densities have both shifted to the right and narrowed substantially. The very significant compression of the bear state density shows again that the possibility of a large fat left tail event has been truncated as the election day has passed. While there is still uncertainty on the final outcome of the election, as of this writing, it seems that the markets are implying little further uncertainty or unexpected events over the next month. It is not the place for us to express our own views on the matter, but to the extent that market participants have the opposite view on the possibility of more uncertainty, we hope the quantification in this section is helpful in creating better portfolio postures if indeed the market consensus turns out to be wrong.

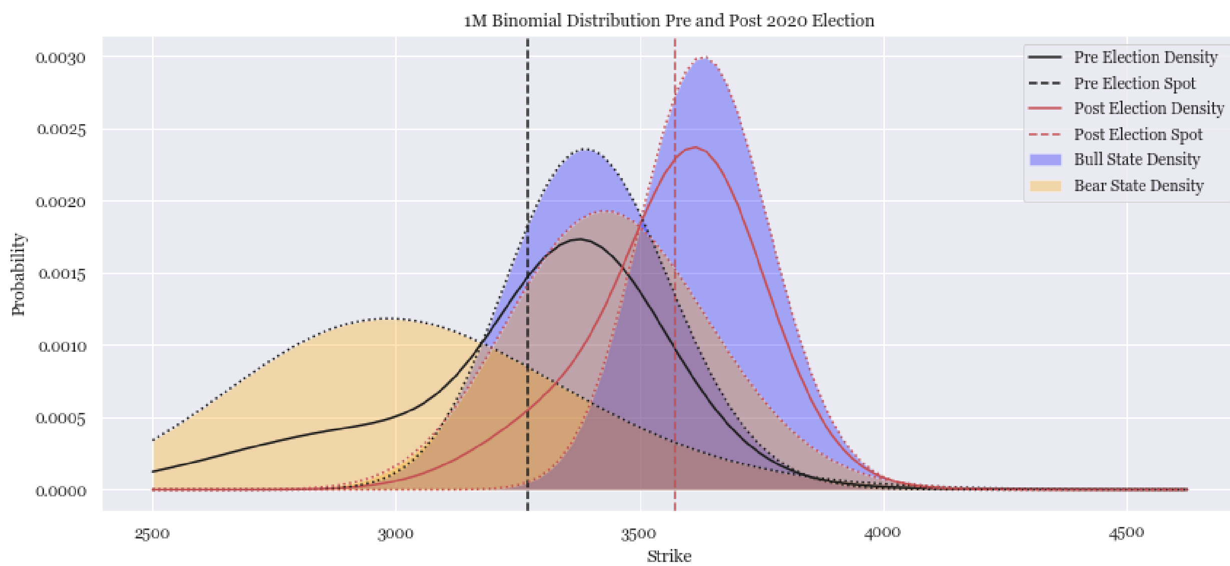

Exhibit 9: 1M Densities Pre and Post 2020 Presidential Election
Source: LongTail Alpha, Bloomberg, OptionMetrics

What Can We Do About It? How Bimodality Can Affect Portfolio Construction

The analysis discussed so far has several implications for portfolio construction that we will illustrate with some examples.

First, for investors who are only concerned with the tradeoff between expected returns and volatility, which is one of the core assumptions in mean variance asset optimizations, then switching out a unimodal distribution of returns for a bimodal distribution will have no impact on the optimal allocation. But, as discussed in Bhansali [2013], to the extent that investors believe tails are important in the portfolio construction process, using a bimodal distribution to model risky asset returns will reduce the optimal allocation of the risky asset in a portfolio.

Exhibit 10 shows the statistics for a unimodal and a bimodal distribution. The bimodal distribution takes the same σ_A, σ_B parameters as the distribution from 11/11/2020 in Exhibit 8 and its μ_A, μ_B parameters are selected to match the annualized return of the S&P 500 index from 01/01/1985 to 10/31/2020. The unimodal distribution is constructed to have the same annualized return and volatility as the bimodal distribution. Notice that the skewness and kurtosis of the bimodal distribution shows that it has a fatter left tail.

	Unimodal Distribution	Bimodal Distribution
P(Bull)	1	0.64
mu(Bull)	0.63%	2.61%
sigma(Bull)	5.26%	3.66%
mu(Bear)	NaN	-2.88%
sigma(Bear)	NaN	6.02%
Annualized Expected Return	9.29%	9.29%
Annualized Volatility	18.36%	18.36%
Skewness	0.16	-0.46
Kurtosis	3.04	3.40

Exhibit 10: Unimodal and Bimodal Distribution Statistics For Asset Allocation Exercise
Source: LongTail Alpha, Bloomberg, OptionMetrics

Next, in order to determine an optimal allocation to risky assets like stocks, we need to assume a utility function of returns, which simply means defining investors' preferences over increases and decreases of their wealth. Exhibit 11 (a) shows the optimal portfolio allocation for both the unimodal and bimodal distribution using a quadratic utility function designed to focus on the tradeoff between expected returns and volatility. Since both distributions have the same annualized return and volatility, and the quadratic utility function ignores the impact of higher moments on preferences by construction, we can see that the optimal allocations are identical.

Because the mean variance utility function is only concerned with the first and second moments of the distribution, it does not incorporate preferences coming from either the left or the right tail of the distribution. To illustrate this point we will look at two games. In game one, investors have to decide between a guaranteed +5% return and a coin toss between receiving 0% or +10%. In game two, the mirror image of game one, investors decide between a guaranteed -5% loss and the coin toss between losing 0% or -10%. In both games, most investors would pick the sure thing because the coin tosses have an additional element of risk. Under a mean variance framework, the perceived benefit of locking in the gain is the same as the perceived benefit of locking in the loss. However, most investors would agree that the guaranteed loss in game two is much more valuable than the guaranteed gain in the first game, i.e. that negative tails matter. This is consistent with the behavior of option participants that bid up the put skew of risky assets. Exhibit 11 (b) incorporates these preferences by looking at the optimal allocations using a CRRA ("constant relative risk aversion") utility function. In contrast to Exhibit 11 (a), we can see that the negative skewness and the higher kurtosis of the bimodal distribution reduces the optimal allocation to the S&P500 index by 4%.

	Unimodal Distribution	**Bimodal Distribution**
Optimal Weight to S&P 500 Index	0.67	0.67
Optimal Weight to Cash	0.33	0.33
Portfolio Annualized Return	6.27%	6.27%
Portfolio Annualized Volatility	12.39%	12.39%
Portfolio Skewness	0.16	-0.46
Portfolio Kurtosis	3.04	3.40

(a) The utility function used to construct the optimal allocation is of a quadratic form $u(r) = r - \eta r^2$where r is the corresponding return on the portfolio consisting of the S&P500 index and cash yielding zero. This utility function creates a mean variance optimization problem and in this example $\eta = 2$.

	Unimodal Distribution	**Bimodal Distribution**
Optimal Weight to S&P 500 Index	0.7	0.66
Optimal Weight to Cash	0.3	0.34
Portfolio Annualized Return	6.49%	6.14%
Portfolio Annualized Volatility	12.82%	12.13%
Portfolio Skewness	0.16	-0.46
Portfolio Kurtosis	3.04	3.4

(b) The utility function used to construct the optimal allocation is a power utility function $u(r) = \frac{(1+r)^{1-\eta}-1}{1-\eta}$ and in this example $\eta = 4$.

Exhibit 11: Optimal Allocation for Quadratic And CRRA Utility Functions Under Unimodality And Bimodality
Source: LongTail Alpha, Bloomberg, OptionMetrics

Another way portfolio construction is impacted is if we believe that some of the market implied distribution estimates are inaccurate. For example, suppose the risk-neutral estimate of 64% probability of being in the good state over the next month was too high, and that the true probability was closer to 50%. Further, let us assume that the outcomes in the good state and bad state were unchanged from what is shown in Exhibit 10, i.e. the expected returns and volatilities were accurate. How would we implement this view?

A reduction in the probability reduces the weighted average expected return since the bad state expected return is lower. Similarly, the volatility of the full distribution would now have a higher value. Exhibit 12 shows the changes in the bimodal distribution statistics when moving to a 50% probability of being in a bull state. The net effect on the optimal portfolio, which we can see in Exhibit 13 (a), is that the allocation to the S&P500 index is reduced by 64%.

	Bimodal Distribution	**Bimodal Distribution with P(Bull)=0.5**
P(Bull)	0.64	0.5
mu(Bull)	2.61%	2.61%
sigma(Bull)	3.66%	3.66%
mu(Bear)	-2.88%	-2.88%
sigma(Bear)	6.02%	6.02%
Annualized Expected Return	9.29%	0.26%
Annualized Volatility	18.36%	19.44%
Skewness	-0.46	-0.34
Kurtosis	3.4	3.08

Exhibit 12: Bimodal Distributions with Decreased Bull State Probability
Source: LongTail Alpha, Bloomberg, OptionMetrics

Given that we have a higher expectation of volatility and lower expectation of return than the option implied values, we can do a number of things. As discussed in Bhansali [2013], we would see the value of long volatility strategies, i.e. via the purchase of options. To achieve this, we could simply implement long put option strategies on the existing asset allocation, or replace some of the long equity position with call options. Exhibits 13 (b) and (c) show the optimal portfolio allocations when including a put option or a call option to the portfolio. This naturally shows the benefits conferred by including options in more robust portfolio construction in the presence of uncertainty about future states of the world.

	Bimodal Distribution	Bimodal Distribution with P(Bull)=0.5
Optimal Weight to S&P 500 Index	0.66	0.02
Optimal Weight to Cash	0.34	0.98
Portfolio Annualized Return	6.14%	0.01%
Portfolio Annualized Volatility	12.13%	0.38%
Portfolio Skewness	-0.46	-0.34
Portfolio Kurtosis	3.40	3.08

(a) Optimal portfolio consists of the S&P500 index and cash yielding zero.

	Bimodal Distribution	Bimodal Distribution with P(Bull)=0.5
Optimal Weight to S&P 500 Index	0.67	0.12
Optimal Weight to Cash	0.2	0.39
Optimal (Notional) Weight to 5% OTM Put	0.13	0.49
Portfolio Annualized Return	6.99%	4.02%
Portfolio Annualized Volatility	11.88%	2.30%
Portfolio Skewness	-0.29	1.74
Portfolio Kurtosis	3.06	8.93

(b) Optimal portfolio also includes a one month 5% OTM put on the S&P500 index.

	Bimodal Distribution	Bimodal Distribution with P(Bull)=0.5
Optimal Weight to S&P 500 Index	0.62	0
Optimal Weight to Cash	0.12	0.48
Optimal (Notional) Weight to 5% OTM Call	0.26	0.52
Portfolio Annualized Return	7.36%	2.96%
Portfolio Annualized Volatility	12.17%	2.50%
Portfolio Skewness	-0.14	3.98
Portfolio Kurtosis	3.6	22.65

(c) Optimal portfolio also includes a one month 5% OTM call on the S&P500 index.

Exhibit 13: Optimal Allocations for Bimodal Distributions

Source: LongTail Alpha, Bloomberg, OptionMetrics

Conclusion

We use a simple mixture distribution framework based on five parameters to estimate the probability distributions of good and bad, or bull and bear states, as reflected in options markets. Our main findings are that implied return distributions change significantly during normal and stressed market situations, and in particular the uncertainty in both distributions also changes significantly during market stresses. We apply this to both the 2016 US Presidential election, and also to the 2020 Presidential election. Our practical conclusions are intuitive and logical: investors faced with highly unpredictable states can manage their portfolios more effectively by dynamically allocating between risky and riskless assets and also by using options. This illustrates that when Mr. Market is behaving erratically, smart investors can avail of the tools from quantitative finance and derivatives markets to create attractive asymmetries in their portfolio.

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IMPORTANT DISCLOSURES

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